1. Work out the characters of $GL_2(\mathbb{F}_q)$.
   - The remaining ones we didn't see in the seminar, that is. And fill in the details.

   E.g. Show $\mathrm{Ind}_B^G \lambda$ is irreducible iff for $\lambda = (\lambda_1, \lambda_2)$ we have $\lambda_1 \neq \lambda$. Otherwise show we can decompose $\mathrm{Ind}_B^G \lambda = \lambda \circ \det \oplus St_\lambda$, where $St_\lambda = St \otimes \lambda$ and St is the q-dim complement of the regular rep.
   - Then do the reps $\mathrm{Ind}\, \theta$, $\theta : \mathbb{F}_{q^2} \longrightarrow \mathbb{C}^\times$

2. Work out the $SL_2(\mathbb{F}_q)$ Deligne-Lusztig varieties using both definitions. In particular, first determine what $B = U \rtimes T$ should be when T is the non-split torus in SL_2.

3. Using the duality $X(T) \times Y(T) \longrightarrow \mathbb{Z}$, or rather $X(T) \simeq \mathrm{Hom}(Y(T), \mathbb{Z})$. Deduce

$$\mathrm{Hom}(Y(T), (\mathbb{Q}/\mathbb{Z})_{p'}) \simeq X(T) \otimes (\mathbb{Q}/\mathbb{Z})_{p'}.$$

Then write down the bijection

$$\mathrm{Irr}(T^F) = T^{*F}$$

we saw in the seminar, for T^* dual torus of T.

④ $G^F \times T^F \curvearrowright Y$ (the Y given in the seminar)

⑤ We consider characters as group homomorphisms
$$\theta : T^F \longrightarrow \overline{\mathbb{Q}}_\ell^\times,$$
and cohomology as $\overline{\mathbb{Q}}_\ell$-vector spaces. Why is this the same as the complex representation theory. Of course, one may identify, non-canonically,
$$\overline{\mathbb{Q}}_\ell \simeq \mathbb{C},$$
but this is a poor reason.