

Deligne Lusztig Learning Seminar Exercises

Week 7

Tom Goertzen

April 9, 2025

1. Show that the general linear group $\mathrm{GL}_n(k)$ together with a Steinberg map F is self dual, where $k = \mathbb{F}_q$.
2. Similiar investigate the duals of other groups: $\mathrm{SL}_n, \mathrm{PGL}_n, \mathrm{PSL}_n, \mathrm{SO}_n, \mathrm{Sp}_{2n}$.
3. What does the main theorem tell us for $\mathrm{GL}_2(k)$ on semisimple conjugacy classes coming from the non-split F -stable maximal torus? Try to compute some character values of the Steinberg character on semisimple conjugacy classes.
4. Parameterise the unipotent characters of $\mathrm{SL}_2(k)$. Why is this situation different from $\mathrm{GL}_2(k)$?